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Autonomous Ring Formation for a Planar Constellation of Satellites

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Introduction

MULTIPLE-SATELLITE rings have been considered recently for a variety of mission applications. These applications center on global point-to-point communications, such as Teledesic and Iridium.[1] The formation and stationkeeping of such large constellations pose new and interesting problems in orbital dynamics.

To ensure global coverage, the intersatellite spacing must be maintained. Small altitude errors will, if uncorrected, result in phasing drifts and clustering of the satellites. There is a requirement, therefore, to ensure that uniform intersatellite spacing is maintained. To control the dynamics of each satellite individually from a ground station would be both complex and expensive. Therefore, an autonomous system is preferable, resulting in lower operational costs and greater operational flexibility.

Such autonomous stationkeeping has been developed for single satellite platforms.[2] However, with multiple satellite rings the system to be controlled must be considered as the entire, collective ensemble of satellites. As such, the orbit control problem is significantly more complex as a large number of individual satellites must be controlled simultaneously.

In this Note a novel, autonomous ring formation and stationkeeping method is considered. Using simple analytic commands, a "loose" ring of satellites can be formed into a perfect ring with uniform intersatellite spacing. The method has analogies with emergent behavior seen in recent studies of nonlinear systems. That is, a set of simple rules is used to generate complex, emergent behavior that is not designed into the system. For this problem analogies may be drawn with the formation of crystal lattices through the minimization of free energy in the system. The satellite ring formation problem is seen to be somewhat similar.

System Dynamics

The dynamics of a system of N satellites will be considered in orbit around a point mass Earth, Fig. 1. The dynamics of the system may then be represented by a system of $2N$ equations of motion, viz.,

$$\left.\begin{aligned} \ddot{r}_i - r_i\dot{\theta}_i^2 &= -\frac{\mu}{r_i^2} + a_{ri} \\ r_i\ddot{\theta}_i + 2\dot{r}_i\dot{\theta}_i &= a_{\theta i} \end{aligned}\right\} \qquad (i = 1, \ldots, N) \tag{1}$$

where a_{ri} and $a_{\theta i}$ are radial and transverse low-thrust control accelerations assumed to be available from onboard thrusters. It is clear that in the open-loop case this system of equations possesses a particular desired solution (r_i^*, θ_i^*) given by

$$\left.\begin{aligned} r_i^* &= \tilde{r} \\ \theta_i^* &= \tilde{\omega}t + 2i\chi \end{aligned}\right\} \qquad (i = 1, \ldots, N) \tag{2}$$

where $\tilde{r}$ is the operational radius of the ring, χ is the vertex half-angle of the N polygon defined by the ring, and $\tilde{\omega}$ is the Keplerian angular velocity at the operational radius. This solution represents the nominal ring with perfect intersatellite spacing.

To investigate ring formation and stationkeeping the $2N$ equations of motion will be linearized relative to the nominal ring. This may be achieved by defining new variables,

$$\begin{aligned} \phi_i &= \theta_i - \tilde{\omega}t \\ l_i &= r_i - \tilde{r} \end{aligned} \tag{3}$$

where the subscript is now understood to run from 1 to N. The linearized system of equations may now be written in the new variables as

$$\begin{aligned} \ddot{l}_i - 2\tilde{\omega}\tilde{r}\dot{\phi}_i - 3\tilde{\omega}^2 l_i &= a_{li} \\ \tilde{r}\ddot{\phi}_i + 2\tilde{\omega}\dot{l}_i &= a_{\phi i} \end{aligned} \tag{4}$$

In linearizing it has been assumed that $l_i/\tilde{r} \ll 1$ and that $\dot{\phi}_i \ll 1$ but not that ϕ itself is small. This set of linear equations may now be used to generate the controls required for ring formation.

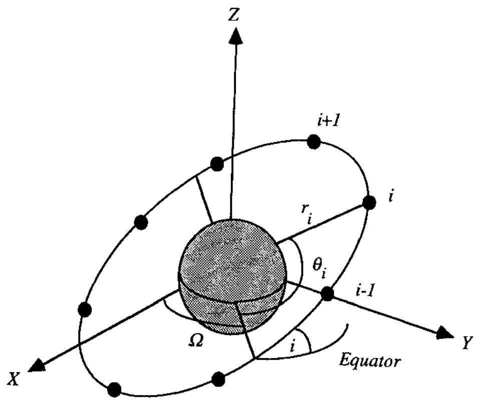

Fig. 1 Schematic geometry of an N-satellite ring.

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Ring Formation

To achieve emergent formation of the ring, Lyapunov's theorem will be used along with a scalar artificial potential function. The potential function method has been developed for other spacecraft control applications.[3,4] To form the ring each satellite will experience a repulsive acceleration in the azimuthal direction due to the other $N-1$ satellites in the ring. Then over several orbital periods, uniform intersatellite spacing will emerge. To generate these accelerations the ith satellite will observe the other $N-1$ satellites as regions of high artificial potential.

It is clear that the required potential must be periodic so that

$$V(\phi+2k\pi)=V(\phi), \qquad k=0,1,2,\ldots, \text{ integer} \tag{5}$$

In principle, a Fourier series may be used to generate suitable periodic functions. However, for ease of illustration the following function will be used:

$$V_i=\lambda_i\sum_{j=1,j\neq i}^{N}\sec^2(\phi_{ij}), \qquad \phi_{ij}=\frac{1}{2}[\phi_i-(\phi_j-\pi)] \tag{6}$$

This function generates singular points of unbound potential when the ith satellite occupies the same azimuthal location as any of the other $N-1$ satellites. The potential falls rapidly away from these locations, yielding a repulsive intersatellite acceleration. The potential of the entire system is then given by

$$\tilde{V}=\sum_{i=1}^{N}V_i \tag{7}$$

The equilibrium configuration of the system will then be achieved when total potential of the system is minimized, viz.,

$$\frac{\partial\tilde{V}}{\partial\phi_{ij}}=0, \qquad \forall i,j \tag{8}$$

This is analogous to minimum energy configurations for crystal lattices.[5] For a periodic system it is clear that a minimum energy configuration is possible when

$$\phi_i-\phi_j=(i-j)\frac{2\pi}{N} \tag{9}$$

This configuration corresponds to the desired uniform intersatellite spacing.

The potential will now be augmented with kinetic terms to ensure that each satellite remains in its azimuthal slot once the ring has been formed. To this end, the total potential will now be written as

$$\tilde{V}=\frac{1}{2}\sum_{i=1}^{N}\left(\dot{\phi}_i^2+\dot{l}_i^2\right)+\sum_{i=1}^{N}V_i \tag{10}$$

The time rate of change of this function is then given by

$$\frac{\mathrm{d}\tilde{V}}{\mathrm{d}t}=\sum_{i=1}^{N}(\dot{\phi}_i\ddot{\phi}_i+\dot{l}_i\ddot{l}_i)+\sum_{i=1}^{N}\dot{V}_i \tag{11}$$

where

$$\dot{V}_i=\lambda_i\sum_{j=1,j\neq i}^{N}(\dot{\phi}_i-\dot{\phi}_j)\sec^2(\phi_{ij})\tan(\phi_{ij}) \tag{12}$$

This expression may be simplified by using the following identity, valid for antisymmetric functions,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\dot{\phi}_i-\dot{\phi}_j)\sec^2(\phi_{ij})\tan(\phi_{ij})$$

$$=2\sum_{i=1}^{N}\dot{\phi}_i\sum_{j=1,j\neq i}^{N}\sec^2(\phi_{ij})\tan(\phi_{ij}) \tag{13}$$

Therefore, the time rate of change of the potential of the system may be written as

$$\frac{\mathrm{d}\tilde{V}}{\mathrm{d}t}=\sum_{i=1}^{N}\dot{l}_i\ddot{l}_i+\sum_{i=1}^{N}\dot{\phi}_i\left\{\ddot{\phi}_i-2\lambda_i\sum_{j=1,j\neq i}^{N}\sec^2(\phi_{ij})\tan(\phi_{ij})\right\} \tag{14}$$

For the minimum energy configuration to be formed it is necessary that the time rate of change of the potential of the system is negative definite. This is clearly analogous to cooling a crystal system for lattice formation.

Substituting for the radial and transverse accelerations from Eqs. (4), one can show that the following controls render the rate of change of the system potential negative definite,

$$\begin{aligned} a_{li}&=-\kappa_{1i}\dot{l}_i-2\tilde{\omega}\tilde{r}\dot{\phi}_i-3\tilde{\omega}^2 l_i\\ a_{\phi i}&=-\kappa_{2i}\tilde{r}\dot{\phi}_i+2\tilde{\omega}\dot{l}_i+2\tilde{r}\lambda_i\sum_{j=1,j\neq i}^{N}\sec^2(\phi_{ij})\tan(\phi_{ij})\end{aligned} \tag{15}$$

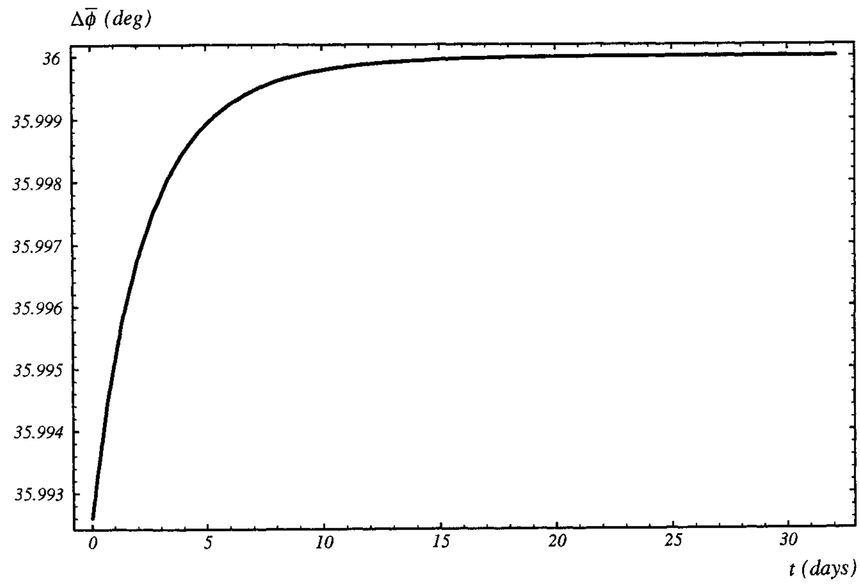

Fig. 2 Evolution of the mean intersatellite spacing during ring formation [$\kappa_{1i}=\kappa_{2i}=10^{-3}$, $\lambda_i=10^{-11}$ ($i=1,\ldots,N$)].

where the constants κ and λ are chosen to generate the desired transient response. Using these controls the rate of change of the potential of the system is given by

$$\frac{d\tilde{V}}{dt} = -\sum_{i=1}^{N} \kappa_{1i} \dot{l}_i^2 - \sum_{i=1}^{N} \kappa_{2i} \dot{l}_i^2 \tag{16}$$

which is clearly negative definite as required.

Implementation

The method will now be implemented in a simple numerical example. A single ring of 10 satellites will be considered at an operating altitude of 400 km. The nominal spacing for the satellites will then be 36 deg. Each satellite will be perturbed by a random amount, up to 10 km in the azimuthal direction. It will be assumed for ease of illustration that there are no radial positioning errors in the system. Such errors can be easily accommodated through minor modifications to the method.

The controls defined by Eqs. (15) generate small accelerations in the radial and transverse directions that generate an azimuthal drift in each of the 10 satellites. As each satellite moves, its motion perturbs the other nine satellites, leading to a complex nonlinear interaction. However, due to the damping terms in the controls, the potential is monotonically decreasing through these interactions. It should be noted that the potential function of any single satellite is not guaranteed to decrease. Only the total potential of the system will monotonically decrease. Therefore, the partitioning of potential between satellites will vary owing to the nonlinear intersatellite interactions.

The mean intersatellite spacing is defined by the following function:

$$\Delta\bar{\phi} = \frac{1}{N} \sum_{i=1}^{N-1} \phi(i+1) - \phi(i) \tag{17}$$

It can be seen from Fig. 2 that a mean spacing of 36 deg is indeed achieved through the use of these controls. This corresponds to the "minimum energy" configuration of the system. This configuration is then stable against perturbations to the ring. For example, if one satellite in the ring were to fail, the ring would then autonomously reform to generate uniform spacing with the remaining $N - 1$ satellites and hence uniform coverage. Similarly, if an on-orbit spare within the ring is activated to replace a failed satellite, the ring will again autonomously reform. Such an autonomous capability may greatly reduce ground segment work loads.

Conclusions

A method has been investigated that allows the autonomous formation of a ring of satellites. The method uses information on the intersatellite spacing to generate low-thrust radial and transverse control accelerations. Using the concept of potential functions, the uniform ring is seen as a minimum energy configuration of the system. The control accelerations ensure that the potential function of the entire system monotonically decreases so that this minimum energy configuration is achieved from any initial configuration. It is believed that such autonomous methods may provide significant operational advantages for future multisatellite rings for global point-to-point communications.

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Orbital Strategies Around a Comet by Means of a Genetic Algorithm

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I. Introduction

THE mission ROSETTA is planned to send a spacecraft to a comet, to rendezvous and explore its features and to land on the surface and take samples. This Note refers to a problem encountered in a previous study[1,2] of computing a complicated sequence of maneuvers to fly over five candidate landing sites while subject to several constraints. Despite the fact that it is initially acceptable to employ classical orbital mechanics approximating the gravitational field as spherical, the dimensionality of the problem and the numerous constraints frustrated an efficient computer algorithm to handle five sites. The use of a genetic algorithm appears, however, to provide a very satisfactory solution.

II. Close Observation of a Comet

A. Specification of the Required Strategy and Constraints

A strategy of maneuvers is required to pass over up to five candidate landing sites at an altitude of 5 km and accomplish the sequence within 20 days subsequent to the following constraints: 1) communication to Earth must not be interrupted by an occultation, 2) orbits should be such that impact does not occur if a maneuver fails, 3) the normal to an orbit plane must not be too close to the direction from spacecraft to Earth in order to preserve a Doppler radiometric ground-based measurement, 4) the landing zones must be illuminated at flyover, and 5) viewing should occur from within 30 deg of the surface normal.

B. Orbital Mechanics

The motion of the comet nucleus is to be regarded as spinning and nutating, although in this Note only spin has been assumed. The extension of the computer program to include nutation would be only a minor modification, viz., preintegration and storage of the Euler equations of rotational motion for a rigid body. As in the earlier study, an irregularly shaped nucleus has been approximated as an ellipsoid, but this could be generalized by means of a representation in terms of a series of spherical harmonics.

As a result of the rotation of the comet nucleus, the specification of a given flyover time over an identified site implies a position vector in nonrotating (ecliptic) axes. Thus, if five flyover times are specified for a given sequence of sites, the trajectory around the comet must pass through five known points. This condition can be satisfied by five maneuvers, and the restriction has been accepted (not necessary) that there is one maneuver in each interval before a flyover time. It follows that for each flyover only the following parameters are necessary: 1) site to be visited and hence a position vector in rotating body axes of the comet nucleus, 2) time of maneuver preceding a given flyover, 3) time of flyover, and 4) orbit integer (+1 or −1) to indicate whether an orbit connects two position vectors by traversing over the subtended angle or 360 deg less than angle.

Further explanation is, however, necessary with respect to the calculation of the orbits passing through the flyover points and starting from given initial conditions. Apart from the use of several standard formulas of conic orbits, a special treatment is needed of Lambert's theorem (Ref. 3, Sec. 7.4), which states that if an orbital transfer occurs from position vector r_1 to r_2, then the time of transfer t depends

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